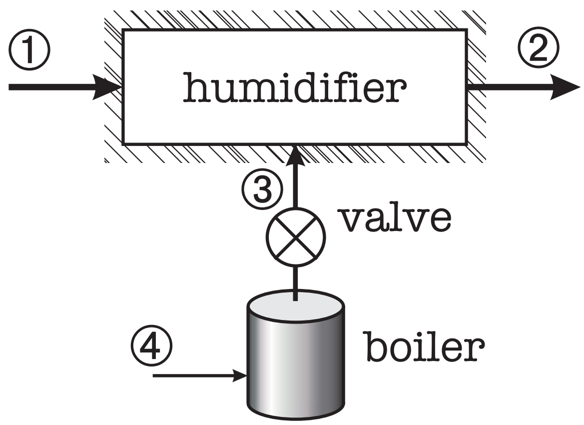
Question 4 (20 marks)

Moist air enters an adiabatic humidifier at state 1 at $\mathbf{20\ °C}$ and **10%** relative humidity with a volumetric flow rate of $\mathbf{0.25\ m^3/s}$. The air leaves the humidifier at state 2 at $\mathbf{22.5\ °C}$ and **70%** relative humidity. The change in the condition of the moist air is brought about by the injection of steam at state 3 from a boiler. The boiler is supplied with $\mathbf{22.5\ °C}$ water at state 4. An adiabatic valve between the boiler and the humidifier causes the steam pressure to drop from boiler pressure to the humidifier pressure of $\mathbf{101.325\ kPa}$.

i) determine the mass flow rate $[\boldsymbol{kg/hr}]$ of the water at state point 4
ii) determine the heat transfer rate $[\boldsymbol{kW}]$ to the boiler
iii) determine the pressure $[\boldsymbol{kPa}]$ in the boiler when the temperature of the superheated steam is $\mathbf{300\ °C}$



We can use either the psychrometric chart or the controlling psychrometric equations to determine the properties at each state.

Part i)
First we note that since the mass of air at state 1 is equivalent to the mass of air at state 2

$$\dot{m}_{a,1} = \dot{m}_{a,2} = \dot{m}_a$$

and from Table A-4, $P_{sat,1}(20\ °C) = 2.339\ kPa$ and $P_{sat,2}(22.5\ °C) = 2.754\ kPa$.

Performing a mass balance for the water over the entire system we get

$$\dot{m}_a\omega_1 + \dot{m}_{w,4} = \dot{m}_a\omega_2 \qquad \Rightarrow \qquad \dot{m}_{w,4} = \dot{m}_a(\omega_2 - \omega_1)$$

$$\omega = 0.622\left(\frac{\phi P_{sat}}{P - \phi P_{sat}}\right)$$

$$\omega_1 = 0.622\left(\frac{0.10 \times 2.339\ kPa}{101.325\ kPa - 0.10 \times 2.339\ kPa}\right) = 0.00144\ \frac{kg_{H_2O}}{kg_{air}}$$

$$\omega_2 = 0.622\left(\frac{0.70 \times 2.754\ kPa}{101.325\ kPa - 0.70 \times 2.754\ kPa}\right) = 0.01206\ \frac{kg_{H_2O}}{kg_{air}}$$

The volumetric flow rate must be converted to mass flow rate as

$$v_a = \frac{R_a T_1}{P_{a,1}} = \frac{R_1 T_1}{(P - \phi_1 P_{sat,1})} = \frac{(0.287)(293)}{(101.325 - 0.1 \times 2.339)} = 0.831\ \frac{m^3}{kg}$$

and

$$\dot{m}_a = \frac{\dot{V}_1}{v_a} = \frac{0.25\ m^3/s}{0.831\ m^3/kg} = 0.30\ kg/s$$

$$\dot{m}_{w,4} = \dot{m}_a(\omega_2 - \omega_1) = (0.30\ kg_{air}/s)(0.01206 - 0.00144)\ \frac{kg_{H_2O}}{kg_{air}}$$

$$= 0.003186\ \frac{kg_{H_2O}}{s} = 11.47\ \frac{kg_{H_2O}}{hr} \Leftarrow$$

Part ii)

Performing an energy balance over the system gives us

$$\dot{m}h_1^* + \dot{m}_{w,4}h_{w,4} + \dot{q} = \dot{m}_a h_2^*$$

Solving for $\dot{q}$

$$\begin{aligned}\dot{q} &= \dot{m}_a(h_2^* - h_1^*) - \dot{m}_{w,4}h_{w,4} \\ &= 0.30\ kg/s(53.7 - 24.0)\ kJ/kg - 0.003186\ kg/s(94.425\ kJ/kg) \\ &= 8.61\ kW \Leftarrow\end{aligned}$$

Part iii)

Performing an energy balance over just the humidifier

$$\dot{m}h_1^* + \dot{m}_{w,3}h_{w,3} = \dot{m}_a h_2^*$$

or by noting that $\dot{m}_{w,3} = \dot{m}_{w,4}$

$$\begin{aligned}h_{w,3} &= \frac{\dot{m}_a(h_2^* - h_1^*)}{\dot{m}_{w,4}} \\ &= \frac{0.30\ kg/s(53.7 - 24.0)\ kJ/kg}{0.003186\ kg/s} = 2797\ kJ/kg\end{aligned}$$

From Table A-6, we can find the pressure of superheated steam when $T = 300\ °C$ and the enthalpy is $2796.6\ kJ/kg$

$$P_3 \approx 800\ kPa \Leftarrow$$

Question 3 (20 marks)

A centrifugal compressor is installed in a natural gas pipeline to overcome the line friction pressure drop. The gas, which is **25%** hydrogen and **75%** methane by volume, enters the compressor at **20 °*C*** and **100 *kPa*** and leaves at **200 *kPa***. Assuming a reversible, adiabatic process and that the properties are independent of temperature;

i) determine the outlet mixture temperature, (°C)
ii) determine the work required to drive the compressor, (kJ/kg)
iii) determine the final partial pressures, (kPa)
iv) determine the change in entropy of the hydrogen and the methane. Verify that the overall process is isentropic.

Part i)

Since the volume fractions are given for the gases, we can also derive the mole fractions

$$\frac{V_i}{V} = \frac{n_i}{n}$$

$$X_{H_2} = \frac{n_{H_2}}{n} = 0.25 \qquad X_{CH_4} = \frac{n_{CH_4}}{n} = 0.75$$

The mass fractions can be determined using

$$Y_i = X_i \left[\frac{\tilde{M}_i}{\sum_{i=1}^{2} X_i \tilde{M}_i} \right]$$

$$Y_{H_2} = 0.25 \left[\frac{2.016}{(0.25 \times 2.016) + (0.75 \times 16.043)} \right] = .0402$$

$$Y_{CH_4} = 0.75 \left[\frac{16.043}{(0.25 \times 2.016) + (0.75 \times 16.043)} \right] = .9598$$

Since the compression process is isentropic we know

$$\frac{T_2}{T_1} = \left(\frac{P_2}{P_1} \right)^{(k-1)/k} = \left(\frac{P_2}{P_1} \right)^{R/c_p}$$

But since we do not know k for the mixture, we must calculate

$$(k-1)/k = \frac{R}{c_p}$$

where

$$\begin{aligned} c_p &= Y_{H_2}(c_p)_{H_2} + Y_{CH_4}(c_p)_{CH_4} \\ &= .0402 \times 14.307 + .9598 \times 2.2537 = 2.738\ kJ/(kg \cdot K) \\ R &= Y_{H_2}R_{H_2} + Y_{CH_4}R_{CH_4} \\ &= .0402 \times 4.1240 + .9598 \times 0.5182 = 0.663\ kJ/(kg \cdot K) \end{aligned}$$

Therefore

$$T2 = (20 + 273.2)\ K\ \times\ \left(\frac{200}{100}\right)^{0.663/2.738} = 346.8\ K = 73.6\ ^\circ C$$

Part ii)

We can perform an energy balance on the compressor to find the work requirement of the compressor.

$$h_1 + \dot{w}_c = h_2$$

$$\dot{w}_c = h_2 - h_1 = c_p(T_2 - T_1) = 2.738\ \frac{kJ}{(kg \cdot K)}(346.8 - 293.2)\ K = 146.76\ \frac{kJ}{kg}$$

Part iii)

$$\begin{aligned} P_{H_2} &= X_{H_2}\ P_2 = 0.25 \times 200\ kPa = 50\ kPa \\ P_{CH_4} &= X_{CH_4}\ P_2 = 0.75 \times 200\ kPa = 150\ kPa \end{aligned}$$

Part iv)

The increase in entropy for the hydrogen is given as

$$\begin{aligned} s_2 - s_1 &= c_{p_{H_2}}\ \times\ \ln\left(\frac{T_2}{T_1}\right) - R\ \times\ \ln\left(\frac{P_2}{P_1}\right) \\ &= 14.307 \times \ln\left(\frac{346.8}{293.2}\right) - 4.124 \times \ln\left(\frac{50}{25}\right) \\ &= -0.4565\ \frac{kJ}{kg \cdot K} \end{aligned}$$

$$
\begin{aligned}
\boldsymbol{s_2 - s_1} &= \boldsymbol{c_{p_{CH_4}} \times \ln\left(\frac{T_2}{T_1}\right) - R \times \ln\left(\frac{P_2}{P_1}\right)} \\
&= \mathbf{2.2537 \times \ln\left(\frac{346.8}{293.2}\right) - 0.5182 \times \ln\left(\frac{150}{75}\right)} \\
&= \mathbf{0.0192}\ \boldsymbol{\frac{kJ}{kg \cdot K}}
\end{aligned}
$$

To verify the compressor is isentropic

$$
\begin{aligned}
\boldsymbol{\Delta s} &= \boldsymbol{Y_{H_2}\Delta s_{H_2} + Y_{CH_4}\Delta s_{CH_4}} \\
&= \mathbf{0.0402 \times (-0.4565)}\ \boldsymbol{\frac{kJ}{(kg \cdot K)}} \ + \ \mathbf{0.9598 \times (0.0192)}\ \boldsymbol{\frac{kJ}{(kg \cdot K)}} \\
&= \mathbf{0.00}\ \boldsymbol{\frac{kJ}{(kg \cdot K)}}
\end{aligned}
$$

If $\boldsymbol{\Delta s}$ of the mixture is zero, therefore the compressor must be isentropic.